Discussion (2/1)

- Probability mass function ($P(X=x)$)

e.g.

x	0	1	2
$P(x)$	$1/4$	$1/2$	$1/4$

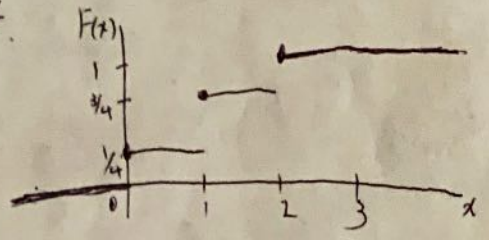
ow $p(x)=0$

$$EX = 0\cdot\frac{1}{4} + 1\cdot\frac{1}{2} + 2\cdot\frac{1}{4} = 1.$$

$$EX^2 = 0^2\cdot\frac{1}{4} + 1^2\cdot\frac{1}{2} + 2^2\cdot\frac{1}{4} = \frac{3}{2}$$

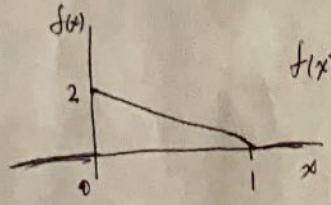
$$VX = EX^2 - (EX)^2 = \frac{3}{2} - 1^2 = \frac{1}{2}$$

- CDF



- PDF (Probability density function) ($f(x)$)

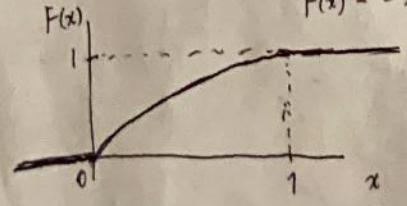
e.g.



$$f(x) = \begin{cases} -2x+2, & 0 \le x \le 1 \\ 0, & \text{ow} \end{cases}$$

$$EX = \int_0^1 -2x+2\,dx = 2\left[-\frac{1}{2}x^2 + x\right]_0^1 = 2\left[-\frac{1}{2}+1\right] = 1$$

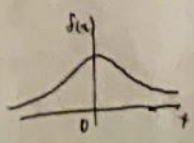
CDF

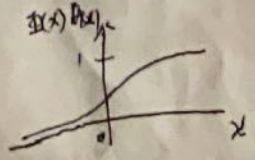


$F(x) = -x^2 + 2x$

- Normal distribution. pdf $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[-\frac{1}{2\sigma^2}(x-\mu)^2\right]$.

standard.





(for $\mu=0$, $\sigma^2=1$)

(Optional)

- Standardization.

$\mathbb{E}(X) = \mu, \quad Var(X) = \sigma^2.$

$$Z = \frac{X - \mu}{\sigma} \quad \mathbb{E}Z = 0, \quad Var(Z) = 1.$$

- e.g. $X \sim N(\mu, \sigma^2)$. $\rightarrow$ $Z \sim N(0, 1)$.

$F(x_2) = \frac{1}{4}$. $\rightarrow$ $\Phi(z_2) = \frac{1}{4}$.

$\int_{-\infty}^{x_2} f(x)\, dx = \frac{1}{4}$. (Substitution rule) $\int_{-\infty}^{z_2} f(\sigma z + \mu)\, \sigma\, dz = \frac{1}{4}$.

- Joint, marginal, Conditional distribution.

[2-d. Discrete case]. (X, Y) PMF $p(x, y)$

Joint: $P(x, y)$ for all Combination of (x, y); (i.e. it's same but using a vector)

Marginal: $P_X(x) = \sum_y p(x, y)$ (n.b. x is a variable);

Conditional: $P_{X|Y}(x|y) = \frac{P(x, y)}{P(y)} = \frac{P(y|x)\, P(x)}{\sum_x p(x, y)} = \frac{P(y|x)\, P(x)}{\sum_x P(y|x)\, P(x)}$

Bayes' rule.

{Be Careful which one is variable, summing over, which is fixed.}

- Expectation (of a function of X, Y)

"A weighted average (or sum) of the possible values with propabilities as weights" $\left\{ \sum_{(x,y)} g(x, y) \cdot P(x, y) \right\}$

Variance $\mathbb{E}(X - \mu)^2$, $Cov(X, Y) = \mathbb{E}[(X - \mu_x)(Y - \mu_y)]$

Do Examples using ~~the first part~~.