Contract and policy design under risk

- Individual decision making typically takes place in a social context.
- Economic institutions affecting individual behavior are themselves subject to management, including
  - property rights
  - enforcement of contracts
  - implementation of policy rules
- Two specific reasons for their significance in risk management:
  - to influence the type and magnitude of individual risk exposure
  - to allow for risk transfers among individuals
- Examples of risk transfer:
  - risk sharing (e.g. sharecropping)
  - insurance protection (e.g., fire or medical insurance)
  - limited liability rules (e.g., bankruptcy protection)
  - social safety nets (e.g., disaster relief managed by government and NGO)

Therefore, the design and implementation of risk transfer schemes among individuals are an important aspect of risk management. In particular, we study efficiency of such schemes using a notion of Pareto efficiency.

Model

Consider a setting:

- n individuals
- m private goods
- A single public good
- Any private good can be transferred from one individual to another
- Individuals obtain utilities by consuming private goods and public good
- Individuals find themselves in one of S random states.

Notations for individuals $i, j \in \{1, \dots, n\}$:

$$
\begin{aligned}
\text{Net output for } i: &\quad x_i = \{x_{1i}, \dots, x_{mi}\} \\
\text{Consumption for } i: &\quad y_i = \{y_{1i}, \dots, y_{mi}\} \\
\text{Transfer from } i \text{ to } j: &\quad t_{ij} = \{t_{1ij}, \dots, t_{mij}\} \\
\text{Public good}: &\quad q \\
\text{Utility function}: &\quad u_i(q, y_i) \\
\text{Random state}: &\quad e \in \{e_1, \dots, e_S\}
\end{aligned}
$$

Note that:

- Private goods (x_i) can be used for consumption, transfer, and production of private goods and public good
- Each net output (x_i) is positive if produced and negative if used for production (i.e. input)
- Self transfer (t_{ii}) is allowed
- Utility functions (u_i) may differ across individuals
- State realizations $\{e_1, \dots, e_S\}$ are mutually exclusive
- Those without subscripts, x, y, and t, imply collections of all the corresponding individual allocations, e.g. $x = \{x_1, \dots, x_n\}$

Our objective is to find an efficient allocation $z = (q, x, y, t)$, which may depend on e. Imagine, specifically, that state-dependent transfer $t_{ij}(e)$ allows a risk transfer across individuals so that we can maximize some aggregate utility or benefits at large.

Feasible allocation

Any allocation (z) must be feasible. In addition to the standard physical constraint, the existence of risk creates another constraint—available information about state. At one extreme, perfect information means that we know the state, i.e. we make an ex-post decision based on a single specific state. At the other extreme, no information means that we make an ex-ante decision based on our subjective probability assessment.

Pareto efficiency

Although efficiency can be defined in many ways, here we use the most standard one in economics, Pareto efficiency.

A feasible allocation is Pareto efficient if there does not exist another feasible allocation that could make one individual better off without making anyone else worse off.

In a 2-person economy ($n = 2$), this is equivalent to an allocation

$$\underset{z}{\operatorname{argmax}}\, u_1(q, y_1) + \gamma u_2(q, y_2)$$

where $\gamma \geq 0$ and z may depend on state e. Intuitively, there is a continuum of Pareto efficient allocations, which is controlled by γ. For example, $\gamma = 0$ means that individual 1 gets everything, which is "efficient" because, by the non-satiation, transferring some from individual 1 to individual 2 necessarily makes individual 1 worse off.

Simple example

Consider a case where:

- Two individuals ($i \in \{1, 2\}$)
- One private good ($m = 1$)
- Ten random states ($e \in \{1, 2, \ldots, 10\}$)
- $u_i(q, y_i) = -\exp(-y_i + i - 2) - \frac{2}{i}\exp(-eq)$
- $y_1 + y_2 \leq B$
- $B = \frac{1}{10}e(8q - 0.4q^2) - 2q$

We may interpret the sole private good as money and B as profit generated through a joint (risky) enterprise characterized by q. (If you prefer more consistent notations, think of the enterprise as jointly established such that $q = x_1 + x_2$. Also, add a third agent whose utility is irrelevant. That is, the physical constraints are $y_i \leq t_{3i}$ for $i \in \{1, 2\}$ and $t_{31} + t_{32} \leq B$.)